American University of Beirut
Math 204
Quiz II (Fall 2014)

Time 70 minutes .

Name: _____ **ID#:** _____

Circle your problem solving section number below:

- **Instructor: Ms Joumana Tannous**

Section 1 @ 1:00 M **Section 2 @ 3:00 M** **Section 3 @ 4:00 M**

- **Instructor: Mrs Maha Itani-Hatab**

Section 4 @ 11:00 Tu **Section 5 @ 8:00 Tu** **Section 6 @ 12:30 Tu**

- **Instructor: Ms. Michella Bou Eid**

Section 7 @ 12:30 Th **Section 8 @ 2:00 Th** **Section 9 @ 5:00 Th**

- **Instructor: Ms Najwa Fuleihan**

Section 10 @ 8:00 Tu **Section 11 @ 12:30 Tu** **Section 12 @ 11:00 Tu**

Page	Grade
2	/ 28
3	/ 27
4	/ 20
5	/ 25
Total	/ 100

1) A fair die is rolled twice and the scores are registered as an ordered pair. Let A be the event that "the sum of the scores is 4", and B be the event that "the score of at least one die is 4".

a. List the outcomes of the events A and B

(6 pts)

$A = \{(1,3), (3,1), (2,2)\}$

$B = \{(4,1), (1,4), (4,2), (2,4), (4,3), (3,4), (4,4), (4,5), (5,4), (4,6), (6,4)\}$

b. Determine whether the events A, and B mutually exclusive.

$A \cap B = \phi \rightarrow$ A and B are mutually exclusive

c. Determine whether the events A and B collectively exhaustive?

$A \cup B \neq S$ since $(2,3)$ is an element in S but not in A nor in B $\rightarrow$ A and B are not collectively exhaustive

2) Nadine has 8 red, 12 green, 7 blue and 3 white beads. In how many ways:

a. can she select 2 red, 1 blue, 4 green and 3 white beads?

(7 pts)

$${}_8C_2 \times {}_7C_1 \times {}_{12}C_4 \times {}_3C_3$$

b. can she make a necklace of 10 beads with exactly 3 blue and 5 green beads?

$${}_7P_3 \times {}_{12}P_5 \times {}_{11}P_2$$

3) A person can order a new car with a choice of 6 possible colors, with or without air conditioning, with or without heated seats, with or without anti-lock brakes, with or without power windows, and with or without a CD player. In how many different ways can a new car be ordered in terms of these options?

(3 pts)

$$6 \times 2 \times 2 \times 2 \times 2 \times 2 = 6 \times 2^5$$

4) On a math test, 5 out of 20 students got an A. If three students are chosen at random without replacement, what is the probability that all three got an A on the test?

(4 pts)

$$\frac{5}{20} \times \frac{4}{19} \times \frac{3}{18} = \frac{{}_5C_3}{{}_{20}C_3}$$

5) How many 6 letter words can you form with exactly 2 A's , 3 B's and 1 C?

(3 pts)

$$\frac{6!}{2!\,3!\,1!}$$

6) Find n if ${}_5P_n = {}_7P_n$

(5 pts)

$$\frac{5!}{(5-n)!} = \frac{7!}{(7-n)!} \rightarrow \frac{(7-n)!}{(5-n)!} = \frac{7!}{5!}$$

$$(7-n)(6-n) = 6 \times 7 \rightarrow 42 - 13n + n^2 = 42$$

$$n^2 - 13n = 0$$

$$n(n-13) = 0$$

$n = 0$ accepted $n = 13$ rejected since $n \leq 5$ and $n \leq 7$

7) Find the probability of selecting a number between 100 and 999 inclusive that has repeated digits?

(4 pts)

$$P(\text{3 digit number with repeated digits}) = 1 - P(\text{3 digit number with no repeated digits}) =$$

$$= 1 - \frac{9 \times 9 \times 8}{9 \times 10 \times 10} = 1 - \frac{648}{900} = \frac{252}{900}$$

8) In how many different ways can 5 boys and 4 girls be seated in a row

a. Without restrictions?

(13 pts)

$9!$

b. If all the girls are sitting next to each other?

$4!\,6!$ or $4!\,5!\,6$

c. If a girl is to sit at each end?

$4P_1 \times 3P_1 \times 7!$ or $4 \times 3 \times 7!$

d. If two boys and one girl want to sit next to each other?

$5C_2 \times 4C_1 \times 3! \times 7!$

e. If the boys and the girls are alternated and the three children (Hani, Jad and Rania) want to sit next to each other?

$3!\,3! \times 4 \times 2$

B G B G B G B G B

9) A small university has applicant pool that is 60% male. The university admits 25% of the male applicants and 75% of the female applicants. One applicant is selected, what is the probability that

(10 pts)

A: admitted
NA: not admitted
M: male F: female

a. the applicant is a female and admitted to the university?

$P(F \cap A) = 0.4 \times 0.75 = 0.3$

b. the applicant is a male given that he is not admitted?

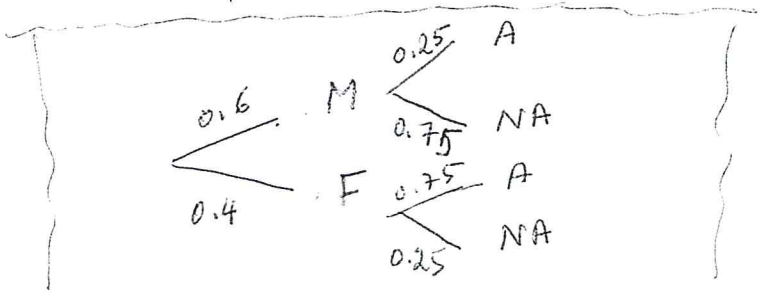
$$P(M/NA) = \frac{P(M \cap NA)}{P(NA)} = \frac{0.6 \times 0.75}{0.6 \times 0.75 + 0.75 \times 0.4}$$

c. the applicant is a female or not admitted?

$$P(F \cup NA) = P(F) + P(NA) - P(F \cap NA)$$
$$= 0.4 + (0.6 \times 0.75 + 0.75 \times 0.4) - 0.4 \times 0.25 = 0.85$$

d. of the admitted students, the applicant is a female?

$$P(F/A) = \frac{P(F \cap A)}{P(A)} = \frac{0.4 \times 0.75}{(0.6 \times 0.25 + 0.4 \times 0.75)} = \frac{0.3}{0.45}$$



0.3. Fourteen people enter the store during a one-hour period. Find the probability that:

(20 pts)

a. exactly 8 persons will make a purchase?

$$P(X=8) = {}_{14}C_8 (0.3)^8 (0.7)^6$$

b. no one will make a purchase?

$$P(X=0) = {}_{14}C_0 (0.3)^0 (0.7)^{14}$$

c. more than 3 persons will not make a purchase?

$$P(Y>3) = 1 - P(Y=0,1,2,3)$$
$$= 1 - \left[{}_{14}C_0 (0.7)^0 (0.3)^{14} + {}_{14}C_1 (0.7)(0.3)^{13} + {}_{14}C_2 (0.7)^2 (0.3)^{12} + {}_{14}C_3 (.7)^3 \right.$$

d. less than 2 persons will not make a purchase?

$$P(Y<2) = P(Y=0,1)$$
$$= {}_{14}C_0 (0.7)^0 (0.3)^{14} + {}_{14}C_1 (0.7)^1 (0.3)^{13}$$

e. at least 2 (A) persons will make a purchase given that at most 4 (B) persons will make a purchase?

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(2 \le x \le 4)}{P(X \le 4)} = \frac{P(X=2,3,4)}{P(X=0,1,2,3,4)}$$

$$= \frac{{}_{14}C_2 (0.3)^2 (0.7)^{12} + {}_{14}C_3 (0.3)^3 (0.7)^{11} + {}_{14}C_4 (0.3)^4 (0.7)^{10}}{{}_{14}C_0 (0.3)^0 (0.7)^{14} + {}_{14}C_1 (0.3)^1 (0.7)^{13} + {}_{14}C_2 (0.3)^2 (0.7)^{12} + {}_{14}C_3 (.3)^3 (.7)^{11} + {}_{14}C_4 (.3)^4}$$

f. In a sample space of 140 persons, what is the expected number of people that will not make a purchase?

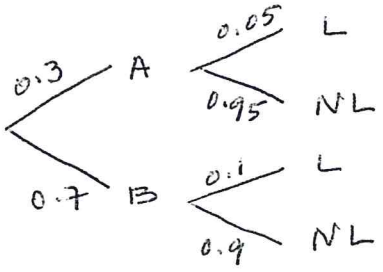
$$140 \times 0.7 = 98$$

g. Let n be the number of persons who enter the store. What is the relation that should be satisfied by n in order to be 99% sure that at least two persons will make a purchase?

$$P(X \ge 2) = 0.99$$
$$P(x = 2, 3, \ldots, n) = 0.99$$
$$1 - P(X=0,1) = 0.99$$
$$P(X=0) + P(X=1) = 0.01$$
$$n C_0 (0.3)^0 (0.7)^n + n C_1 (0.3)^1 (0.7)^{n-1} = 0.01$$
$$0.7^n + n (0.3)^1 (0.7)^{n-1} = 0.01$$

11) Nadia goes to work by one of two routes A or B. The probability of going by route A is 30%. If she goes by route A the probability of being late for school is 5% and if she goes by route B, the probability of being late is 10%.

(13 pts)



a. Find the probability that Nadia is late for school

$P(L) = 0.3 \times 0.05 + 0.7 \times 0.1$

b. Given that Nadia is late for school, find the probability that she went via route A.

$$P(A|L) = \frac{P(A \cap L)}{P(L)} = \frac{0.3 \times 0.05}{0.3 \times 0.05 + 0.7 \times 0.1}$$

c. Find the probability that Nadia went via route A or she is not late for school

$$P(A \cup NL) = P(A) + P(NL) - P(A \cap NL)$$
$$= 0.3 + (0.3 \times 0.95 + 0.7 \times 0.9) - 0.3 \times 0.95$$

d. Find the probability that Nadia is neither late nor going via route B.

$$P(L \cup B)' = 1 - P(L \cup B) = 1 - [P(L) + P(B) - P(L \cap B)]$$
$$= 1 - [(0.3 \times 0.05 + 0.7 \times 0.1) + 0.7 - (0.7 \times 0.1)]$$

12) Given that $P(A \cup B')' = 0.2$, $P(A'/B') = 0.25$ and $P(A'/B) = 0.25$.

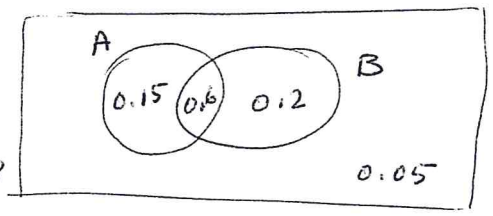
(12 pts)

a. Find $P(B)$.

$P(A \cup B')' = P(A' \cap B) = 0.2$

$P(A'|B) = \frac{P(A' \cap B)}{P(B)} = \frac{0.2}{P(B)} = 0.25$

$P(B) = \frac{0.2}{0.25} = 0.8$



b. Find $P(A \cap B)$.

$P(A \cap B) = P(B) - P(A' \cap B)$

$= 0.8 - 0.2 = 0.6$

c. Find $P(A \cup B)$.

$$P(A'|B') = \frac{P(A' \cap B')}{P(B')} = \frac{P(A \cup B)'}{1 - P(B)} = 0.25$$

$P(A \cup B)' = 0.25 \times 0.2 = 0.05$

$P(A \cup B) = 0.95$

d. Find $P(A)$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.95 = P(A) + 0.2 - 0.6 \rightarrow P(A) = 0.75$

e. Are the events A and B independent?

$P(A) \times P(B) = 0.75 \times 0.8 = 0.6 = P(A \cap B)$

A and B are independent